ANL-ART-167

Inelastic analysis procedure based on the Grade 91 unified viscoplastic constitutive model for ASME implementation

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Applied Materials Division

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Abstract

This report describes a design by inelastic analysis procedure for Grade 91 to be proposed for incorporation in the ASME Boiler and Pressure Vessel Code. The package of proposed additions includes guidance on inelastic modeling and a reference constitutive model for Grade 91 to be used with the existing Section III, Division 5, Subsection HB, Subpart B rules for design by inelastic analysis. The current Code specifies acceptance criteria for design by inelastic analysis but does not provide an acceptable constitutive model for Grade 91 or any of the other Class A materials. The proposed Nonmandatory Appendix would provide such a reference model, greatly easing the use of design by inelastic analysis. As the inelastic methods produce significantly less overconservative designs than the design by elastic analysis rules this proposed Code change could reduce the design and fabrication cost of future high temperature reactors.

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1 Introduction

This report summarizes work on implementing, justifying, and putting into practice a design by inelastic analysis method for Grade 91 (9Cr-1Mo-V) steel. The ultimate goal is to provide designers an ASME Boiler and Pressure Vessel Code approved way to design high temperature, safety-critical Grade 91 reactor components using inelastic analysis. The basis for the design method is the existing design by inelastic analysis provisions in Section III, Division 5 Subsection HB, Subpart B Nonmandatory Appendix-T of the ASME Code. These rules cover two deformation mechanisms: ratcheting strain accumulation and creep-fatigue damage. The current rules assume the designer has access to an inelastic constitutive model that accurately describes the high temperature cyclic deformation of the material. However, the current Code does not provide such a material model for any of the materials, nor does it provide guidance on how to construct such a model.

NRC is currently reviewing the 2017 edition of Division 5 for endorsement through a Regulatory Guide. Future editions of Division 5 will be assessed by NRC so that applicable new rules could be added in the Regulatory Guide. A Regulatory Guide provides guidance to licensees and applicants on implementing specific parts of the NRC's regulations, techniques used by NRC in evaluating specific problems or postulated accidents, and data needed by NRC in its review of applications for permits or licenses. Thus incorporation of inelastic material models into Division 5, Subsection HB Subpart B and their subsequent endorsement through their inclusion in the Regulatory Guide would promote regulatory efficiency on the part of NRC and reduce regulatory uncertainty on the part of the licensees and applicants. For this Division 5 endorsement pathway, the use of alternate inelastic material models would require justifications by licensees and applicants and additional efforts by NRC to evaluate the alternate approaches.

Previous DOE sponsored research developed a model for Grade 91 suitable for use with the Code design by inelastic analysis provisions. This work was described in previous DOE reports [1, 2] and a paper disseminating the model to the high temperature design community [3]. The focus of work this year was on providing general guidance on how to construct a suitable model and on laying the groundwork for incorporating the model into the Code. Chapter 2 of this report discusses the development of such guidance and its planned implementation in the ASME Code.

After discussion with the ASME Section III Working Group on Inelastic Analysis Methods a decision was reached to wait until a substantial portion of the guidance and reference inelastic models were completed before submitting a ballot. The plan is to ballot a package containing the general guidance, a model for Grade 91, and a model for 316H stainless steel. Previous work completed the Grade 91 model, parallel work in a separate work package is developing the 316H model, and this work represents the substantial completion of the general guidance. A package will be assembled for ballot at an upcoming Code Week. Consistent with the current design by elastic analysis rules in Nonmandatory Appendix T, the guidelines and reference models will be included as a nonmandatory Appendix Z of Section III, Division 5, Subsection HB, Subpart B. Appendix A of this report provides the current draft guidance, which is discussed in detail in Chapter 2

Chapter 3 summarizes past work on the Grade 91 inelastic model and provides a summary of the model development and validation process to support balloting the model to ASME Code Committees. This Chapter will form the basis of a background document provided to

ASME along with the proposed Code change. The Appendix reproduces the proposed ASME Code language, while Chapter 3 provides background information on the model. Chapter 2 and 3 combined represent the completed Grade 91 inelastic design method.

The final chapter of the report summarizes the Grade 91 modeling work and describes the remaining steps required to implement the model in the ASME Code.

2 Guidance on inelastic modeling

2.1 Background and requirements

As described in the introduction, the current ASME Boiler and Pressure Vessel Code provides evaluation procedures for designing Class A high temperature reactor components using the results of an inelastic stress analysis. The current Code provisions are acceptance criteria: given the stress, strain, and time history of a given material point (or stress classification line for the ratcheting rules) the criteria provide a series of pass or fail design checks covering the Code ratcheting strain accumulation and creep-fatigue design criteria. Generating that stress/strain/time history requires simulating the transient history of a component using a suitable inelastic constitutive model. The current Code does not provide constitutive models for the Class A materials nor does it provide guidance on how to construct such a model from data. However, the typical data requirements to support the development of an inelastic model are provided in the nonmandatory Appendix HBB-Y.

The current Code is circumspect about what criteria a suitable model should meet. HBB-3212(b) states

> For inelastic analysis required by Subsection HB, Subpart B, appropriate multiaxial stress-strain relationships and associated flow rules shall be used to combine multiaxial stresses and strains.

and HBB-3212(c) notes some special characteristics of the response of Grade 91 steel, notably that it is strongly rate and temperature dependent and that the material exhibits cyclic softening above 1000° F. The Code distinguishes a full inelastic analysis, which includes creep, from a plastic analysis, which accounts only for time-independent plastic deformation, in HBB-3213.23 and HBB-3213.24. HBB-3214.2 describes inelastic analysis in detail. It notes that generally this sort of analysis will combine time independent plastic deformation with time dependent creep deformation. It notes a list of material features that should be captured by a model:

- Strain and cyclic hardening or softening
- Primary creep and creep strain hardening or softening
- The effect of creep on plasticity and vice-versa.

This provision requires that the selected inelastic models be described in the Design Report. The provision also requires the designer to use average material properties in constructing the model, with the exception of analysis to check the structure for buckling. Finally, the provision again notes some of the special deformation characteristics of Grade 91 and notes that adequately modeling these characteristics, in particular the material's large rate sensitivity at low temperatures, may require a unified viscoplastic model that treats creep and plasticity as a single phenomenon. Both this provision and HBB-T-1121 note that for the secondary load criteria inelastic analysis is actually the default method and that the various simplified design by elastic analysis rules may be too conservative at certain locations. At these locations a full inelastic analysis would be required to demonstrate the adequacy of the design.

The current ASME design by inelastic analysis provisions date to the Clinch River Breeder Reactor Project (CRBRP) in the 1970s and early 1980s. The practice of inelastic analysis at that time was summarized in a Welding Research Council (WRC) Bulletin [4]. A good portion of the report deals with the development and implementation of the inelastic constitutive models used in the CRBRP. These models are discussed in a subsequent section.

The report notes that the order and history of simulated transients affects the results of the nonlinear analysis and contains guidelines for selecting a subset of the full plant history to analyze. The suggestion is to replace all the different Level A, B, and C loading cycles with a single worst-case representative cycle from each category, use a simplified method to determine the most conservative ordering of the bounding cycles, and analyze the structure with this bounding composite load cycle. The procedure allows extrapolating creep-fatigue damage from a limited number of load cycle repetitions. These guidelines are aimed at reducing the computational effort, and hence, time required for the analysis. Perhaps surprisingly, the suggestions are still relevant today as the fundamental computational issue has not been addressed – time integration cannot be parallelized. That said, modern computational capabilities vastly exceed those available to the CRBRP and so such aggressive load cycling coarsening is no longer required. In addition, the guidance provided on coarsening the spatial discretization of FE models is no longer relevant as parallel computing can rapidly solve FE problems over fine meshes.

An additional chapter of the WRC report summarizes the capabilities of then-current FE solvers. The description is entirely outdated, though interestingly two of the solvers discussed in the report, Abaqus and ANSYS, are still the dominate commercial analysis packages today. The remaining chapters describe in detail two inelastic calculations used in the CRBRP: the design of the Intermediate Heat Exchanger primary sodium inlet nozzle and the piping design of the Primary Heat Transport System. The examples provide some guidance in historical practice in simplifying the component geometry and transient history and executing the Code rules, but they take the constitutive model as specified and so do not provide guidance on model development.

2.2 Past constitutive models

Past DOE work during the CRBRP developed constitutive models for 2.25Cr-1Mo and 304H stainless steel. These models were considered, at the time, sufficient for use with the Code design by inelastic provisions so it is worthwhile to look at the form of these previous models. The historical development of these model is well-documented in the literature [5, 6, 7, 8, 9] and so this summary focuses on the common details of the models. Of interest, the CRBRP models for 304H and 316H stainless steels, often referred to as the Oak Ridge National Laboratory (ORNL) models, remain implemented in current versions of Abaqus [10].

The models use non-unified forms that additively decompose strain into contributions from elastic deformation, rate independent plastic deformation, time dependent creep deformation, and thermal strain

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_c + \varepsilon_T. \tag{2.1}$$

The elastic model is standard linear elasticity with temperature dependent modulii. The plasticity model uses classical rate independent J_2 flow theory with linear kinematic hardening and a temperature dependent definition of yield stress and kinematic hardening modulus. The creep models uses J_2 flow theory and several options were provided for the creep rate model including power law, exponential, and rational polynomial forms.

Several ad hoc modifications to account for interactions were superimposed on top of this base model. The first modification was a change in the kinematic hardening parameters at the 10th load cycle. This change reflects experimentally-observed shifts in the materials' short- and long-term cyclic hardening responses. In modern models [11] multiple backstress terms would be used to capture these shifts, but the 10 cycle procedure captures the effect while maintaining only a single backstress.

The second modification is the so-called α-reset procedure, thus called because the model theory uses the symbol $\boldsymbol{\alpha}$ to denote the backstress. This modification was required because linear kinematic hardening developed far too large of a backstress causing, for example, a high mean stress in simulations of thermal ratcheting tests, contrary to the available experimental data. This process reduces the value of the backstress during elastic unloading (i.e. during load reversals in cyclic tests) using an ad hoc rule. A final modification is the β-option which couples prior plastic deformation to strain hardening during creep again using an ad-hoc rule.

All of these modifications have been obviated by more modern techniques. However, the focus in the CRBRP models on capturing the interaction of creep and plasticity clearly indicates the importance of handling this interaction in any constitutive model used to execute a Section III design by inelastic analysis [12].

2.3 Proposed guidance

A set of guidelines were developed based on these historical practices and current best practices for design by inelastic analysis. Appendix A reproduces the current draft of this guidance in full. The guidance, along with Grade 91 and 316H material models, will be balloted as a Code change proposal to add the information as a Nonmandatory Appendix to Section III, Division 5, Subsection HB, Subpart B of the ASME Code. By making the guidance and models nonmandatory this approach preserves the current option of allowing the Owner/Operator or designer to select their own material model while still providing better guidance and specific models so that a designer can pick up the Code and immediately begin a design by inelastic analysis. The following sections discuss the proposed guidance point by point.

2.3.1 Objective, requirements, and specification

The proposed Appendix provides guidance on developing inelastic constitutive models and particular models for the Class A materials. The appendix is nonmandatory and refers to the base Code requirements for inelastic analysis contained in Section III, Division 5 HBB-3214.2.

2.3.2 General guidance on modeling

The general goal of a constitutive model is to accurately characterize the average response of a material to time-dependent, cyclic, thermomechanical load. In particular, it must represent time-independent cyclic plasticity, primary creep, the effect of creep on plasticity, and the effect of plasticity on creep.

The model must be valid over the temperature ranges experienced by the component in service. Depending on the specified Service Loadings the temperature may descend below the elevated temperature threshold temperatures specified in HAA-1130-1. Even though the design in that low temperature range is not covered by Subsection HB, Subpart B of the Code, the material model must be valid at those low temperatures and the design analysis must include that portion of the loading. The reason is that plasticity in this low temperature excursion may affect both subsequent high temperature plasticity through strain hardening or softening as well as subsequent creep deformation.

The model must account for the rate sensitivity of the material throughout the expected strain rates experienced by the component in service. These strain rates will not be known a priori. Typical strain rates during hold periods of near-constant loading will be less than $10^{-6}\,\mathrm{s}^{-1}$ while strain rates during transients may exceed $10^{-4}\,\mathrm{s}^{-1}$. The model response must vary smoothly with temperature and rate. Therefore, the model must specify some temperature interpolation scheme and not simply a sequences of parameter definitions at fixed temperatures.

2.3.3 *Key features*

The proposed HBB-Z-1212 describes material features that all models must capture. These aspects of modeling generally apply to all the Class A materials.

2.3.3.1 *Cyclic response*

The key experimental data the model must capture is the stress/strain/time hysteresis of a material undergoing some period load intermixed with hold periods at constant load. The model should be able to accurately represent this response for either stress or strain controlled loading or for mixed loading conditions, i.e. elastic follow up. The experimental data is typically supplied using strain-controlled cyclic tests, i.e. creep-fatigue tests, or stress-controlled cyclic tests, i.e. ratcheting tests.

2.3.3.2 *Batch variation*

As described in Subsection HB, Subpart B, the inelastic constitutive model should represent the average material response. By default, this means the model should represent the average response of all material meeting the material specifications described in Table HBB-I-14.1(a). In general, the variability in material properties can be quite large (c.f. Figure, plotting the yield stress of Grade 91 for many heats of material). However, if the designer can demonstrate that the component will be constructed by material with reduced variability and a different average response then they are free to factor that into the constitutive model. The acceptable models provided in the Appendix reflect the average properties of the material, including all historical heats.

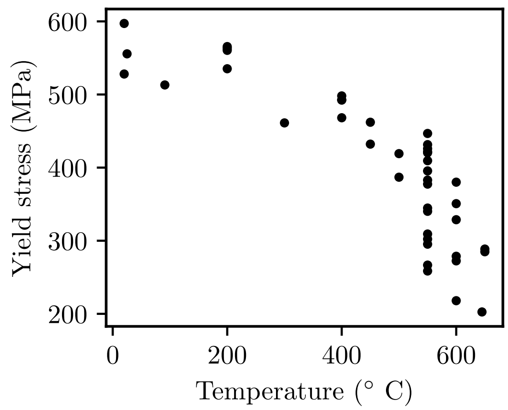

Figure 2.1: Scatter in measured Grade 91 yield stress as a function of temperature, provided to indicate the amount of heat-to-heat material variability expected for the Class A material properties. Scatter in fatigue, creep, and creep-fatigue data is even greater.

2.3.3.3 Accumulated damage

A material model for Code use does not need to represent the degradation caused by cracking, void growth, and other deterioration mechanisms. Damage is tracked separately in the course of the Code design and acceptance calculations and so to include damage effects in the constitutive model would be double-counting the effects of damage. However, there are exceptions to this general principle. Cyclic or strain softening caused by dislocation recovery and microstructural evolution should be captured in the model as these mechanisms represent changes in the material deformation and not the accumulation of unrecoverable damage. Grade 91 is a prime example of a material that exhibits these mechanisms [13].

For the purposes of ratcheting strain accumulation the model should capture tertiary creep which is often caused by damage mechanisms. The model must account for the acceleration in the creep rate in order to accurately characterize the final, accumulated inelastic strain in the material which is the acceptance criteria provided in the Code. For this purpose the model might incorporate a creep damage model.

This guidance could lead to the construction of two inelastic models: one used for creep-fatigue evaluation that does not include damage mechanism and one for ratcheting strain evaluation that includes the damage mechanisms causing tertiary creep. Defining two models in this manner is acceptable. However, it is conservative to use a single model accounting for tertiary creep for both the Code deformation and damage checks.

2.3.3.4 Coupled creep-plasticity

As noted above, it is crucial the model capture the interaction of prior plasticity on creep and prior creep on plasticity. As noted above, historically these interactions were captured using ad hoc models interfacing between traditional rate independent plasticity and rate dependent creep models. A more modern approach is a unified viscoplastic model, which treats creep and plasticity with a single model [14, 15, 11]. While not required, such models can be used to represent creep-plasticity interaction. However, at sufficiently high temperatures creep and plasticity are indistinguishable deformation mechanisms and so a unified viscoplastic approach is required. The draft guidance provides temperatures for each of the Class A materials above which creep and plasticity become indistinguishable. These temperatures are based on past DOE sponsored work [16, 17].

2.3.3.5 Stress multiaxiality

The vast majority of experimental test data is gathered for uniaxial tension or compression loading. However, the stress state in actual components is multiaxial. The constitutive model must extend the uniaxial dataset to multiaxial loading. Typically this will be done through standard theories, i.e. J_2 flow theory, but the results of applying these standard theories should ideally be validated.

2.3.4 Validation

Uniaxial cyclic tests, with and without holds, under strain and stress control, and at a variety of temperatures are the most straightforward means of experimental validation. However, the available cyclic test data typically uses short hold periods, much shorter than the actual holds at constant load in service. Therefore, the model should additionally be validated against long-term creep and stress relaxation test data.

Recognizing that multiaxial test data is scarce the recommendation is to extend the uniaxial data using standard flow theories and then validate the results against multiaxial tests. This might be a cyclic tension-torsion test or some form of pressurized tube test.

2.3.5 Specific material issues

This section summarizes model development and validation issues specific to particular materials. The current draft includes a subsection for each of the Class A materials to permit future expansion but currently the draft only has notes for two materials.

2.3.5.1 2.25Cr-1Mo

2.25Cr-1Mo softens under cyclic load under some loading and temperature conditions within the scope of Division 5. A constitutive model must account for this softening.

2.3.5.2 Grade 91

This subsection reiterates the notes on Grade 91 already described in the base Code: Grade 91 shows both work and cyclic softening and shows rate sensitivity at very low temperatures. The constitutive model must capture these effects.

3 Grade 91 inelastic model

This chapter describes the final Grade 91 inelastic modeling proposed for incorporation into the new Appendix Z. Full details of the dataset, model development, and model calibration are found in past work [1, 3, 2]. This report summarizes the final model and describes the model features and validation process in reference to the guidance described in the previous chapter. Appendix A of this report contains a full mathematical description of the model and lists the model parameters in the form of a revision to the ASME Boiler and Pressure Vessel Code. This chapter will form the basis of the background document provided along with the proposed code change.

3.1 Model features

3.1.1 Model form

The model uses a Chaboche isotropic and kinematic hardening model to capture the details of cyclic plasticity in Grade 91. This model was developed specifically to capture complicated cyclic plasticity response and has been validated to work effectively for a large variety of materials [18, 19, 20, 21, 22, 23, 24, 11]. Because Grade 91 shows cyclic and work softening behavior the standard Voce isotropic hardening model is reversed to provide temperature-dependent isotropic softening.

Grade 91 exhibits an unusual cyclic behavior called anomalous ratcheting [25]. Under fully-reversed stress controlled loading the material will ratchet by accumulating significant tensile strain. This behavior implies tension/compression asymmetry, which should be captured by the constitutive model. The final model uses a non-J_2 flow theory to account for this asymmetry [26].

The model captures the accelerating creep rate in tertiary creep with a combination of this isotropic softening and the static recovery terms of the complete Chaboche model. Both features tend to reduce the value of the flow stress for long hold times, increasing the resulting creep rate. No attempt was made to capture the final stages of tertiary creep, where void growth mechanisms produce a significantly accelerated creep rate. The rational for this decision is outlined in the previous section on model development guidelines.

As noted by the current version of Section III, Division 5, Subsection HB, Subpart B Grade 91 demonstrates a rate sensitive plastic response at relatively low temperatures. In the rate sensitive regime creep and plasticity are essentially indistinguishable. The model captures this effect, and the general interaction of creep and plasticity, using a unified viscoplastic flow rule. However, to capture the model response at lower temperatures the model switches to a rate independent response. A new theory was developed to determine whether at a given temperature and strain rate the model should use a rate independent or rate dependent update. The new theory is based on Kocks-Mecking kinetics [27, 28] that unify temperature and strain rate into a single normalized activation energy. This activation energy can be used to both determine whether a rate dependent or rate independent update should be applied as well as to determine the temperature and rate dependent values of the yield stress, rate sensitivity exponent, and flow viscosity.

The model uses standard flow theory to convert from a uniaxial to a multiaxial response. This is one of the primary weaknesses of the current model as there is essentially no validation

Source	Test types
Asayama and Tachibana[29]	Creep, stress relaxation
Choudhary and Isaac Samuel [30]	Creep
Kim and Weertman [13]	Monotonic, cyclic
Kimura, Kushima, and Sawada[31]	Creep
Koo, Lee, and Kwon [32, 33]	Cyclic, stress relaxation
Latha et al. [34]	Monotonic, creep
Maruyama et al. [35]	Creep
Swindeman [36]	Monotonic
Yaguchi and Takahashi [37, 25, 26]	Monotonic, stress relaxation, cyclic
Zhang and Aktaa [38]	Cyclic
DOE historical data	All types of experiments

Table 3.1: Summary of data sources for Grade 91 steel.

data to support or refute this assumption.

3.1.2 Calibration process

In order to capture the average response of all historical Grade 91 material the model was calibrated to a large experimental data set gathered from past DOE sponsored research, a literature survey, and international databases. Table 3.1 summarizes the type and sources of test data.

The model was calibrated to this data using genetic algorithm optimization. The general approach was to simulate each test and collate the simulation data in the same manner as the test data. For example, for creep tests the result would be a simulated and an experimentally-measured creep curve. The integrated difference between the two curves, normalized by the maximum value, provided the measure of similarity used in the optimization algorithm. This processes was repeated for each test in the database and the results summed to form a global goodness-of-fit metric. Different test types were given different weights in forming the final objective function. Originally, these weights were set to $1/n_{type}$ where n_{type} is the number of tests of a given category, for example the number of creep tests. Later these weights were tweaked based on engineering judgment of which types of test data were more important. These tweaks prioritized cyclic data over long term monotonic testing.

This process was applied to the totality of the data at all temperatures simultaneously. Several temperature control points, shown in the parameter table in Appendix A, were selected and the parameters interpolated linearly or log-linearly between those points. This method of fitting assess the complete, temperature dependent model against the data, rather than looking at just one temperature at a time. The overall approach was designed to ensure the final model represents the average behavior of all the tested Grade 91 material.

3.2 Model validation

Detailed validation of the model was described in the past reports. The model was validated against specialized tests not included in the original fit database including standard

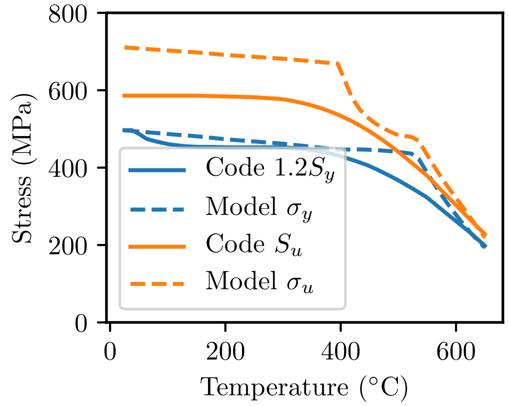

Figure 3.1: Comparison between the ASME Code design values of yield and ultimate stress and the corresponding model predictions.

thermomechanical cyclic tests to assess the model response to non-isothermal loads. A validation of the model against non-uniaxial load was not completed as no such experiment could be identified for Grade 91. This section focuses on comparisons to ASME Code data that demonstrate that the model captures the average material response as indicated by the Code as well as summarizing a few of the important validation tests from previous work.

The ASME Code provides design values of yield strength (S_y) and ultimate strength (S_u) as a function of temperature. The design yield strength is a shift to the actual temperature dependent yield stress data to align the room temperature value with the minimum specified yield stress. An approximate way to undo that shift is to multiply the values of S_y by 1.25. The design values of ultimate stress are close to average properties. Figure 3.1 compares the yield and ultimate stresses from the ASME Code to those generated from the model. The corresponding simulations were run at the ASTM E-21 strain rate of $8.33 \times 10^{-5}\,\mathrm{s}^{-1}$. Except for the ultimate tensile stress near room temperature, where the flow curves used to calibrate the model to data do not extend to the ultimate tensile stress, the model and ASME Code values are in good agreement. The discrepancy in the yield stress between 400° and 525° C is the model moving towards the upper-shelf value of yield stress to capture the available full tensile curves in this temperature range. Figure 3.2 shows the model is still well within the experimental scatter in yield stress.

The ASME Code provides a model for uniaxial deformation for Grade 91 in the form of isochronous stress-strain curves. These curves are generated from a standard, non-unified elastic + rate-independent plasticity + creep model developed in [39]. This model can be used to generate synthetic creep tests for comparison to the viscoplastic model. The ASME model and the full viscoplastic model describe here will not exactly coincide as the unified model attempts to capture both monotonic and cyclic data, whereas the ASME model only captures monotonic load. The additional data considered over the single batch used to calibrate the ASME model also affects the final model calibration. However, good agreement between the ASME model and the viscoplastic model implies the new model is capturing the average material response as described in the current Code.

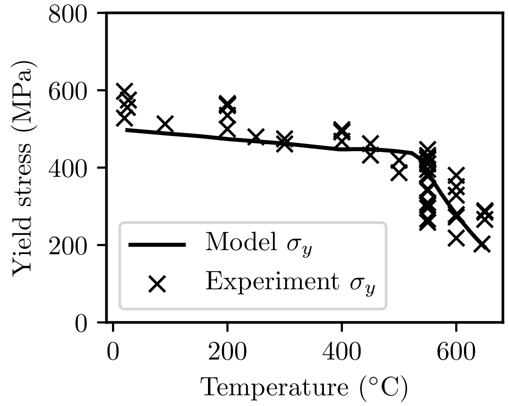

Figure 3.2: The model prediction for yield stress compared to the experimental data.

Figure 3.3a-f compares creep curves generated with the ASME model for deformation to the corresponding model predictions. These plots were generated for temperatures within the scope of Section III, Division 5, Subsection HB, Subpart B and therefore in the creep regime. Each plot represents one temperature. The stresses for each plot are selected to be $\sigma_y/4$, $\sigma_y/2$ and $3\sigma_y/4$ to avoid exceeding the material's ultimate stress at the higher temperatures. The creep deformation is limited to 2% strain as that is the range of applicability using the ASME design provisions. For the low temperature, low stress cases where the creep rate is very small, the total creep time is limited to 50,000 hours and the material will not exceed 2% creep strain even in that period. The loading rate is not a parameter of the ASME model, but is a parameter for the viscoplastic model. This information is not commonly measured or recorded for creep tests and so a loading rate of $1\,\mathrm{MPa/s}$ was assumed. At temperature below 500° C there is substantial disagreement between the ASME and viscoplastic model. *In this regime there is no data* – both models are extrapolating creep test data from high temperatures to lower temperatures. Creep strain rates at realistic component stresses will be very small in this temperature regime and the time spent in this regime for current reactor concepts will be small. At temperatures above 500° C the models agree well within the scatter in the available creep test data.

Unfortunately, the ASME Code does not provide a description of material deformation under cyclic load. A direct comparison to the cyclic data, summarized in past reports, validates the model against the available creep-fatigue (strain-controlled) and ratcheting (stress-controlled) data. As validation several tests were withheld from the fitting database and used to assess the calibrated model's cyclic response. Figure 3.4 provides an example of this validation comparing the model and experiment stress-strain hysteresis data. In the experimental data the load cell did not read zero at the start of the test. The data was shifted by a constant offset to start at zero. Similarly, the ASME Code data does provide a method for validating the non-isothermal response of the model. The past reports validate the model against standard ASTM thermomechanical cyclic tests.

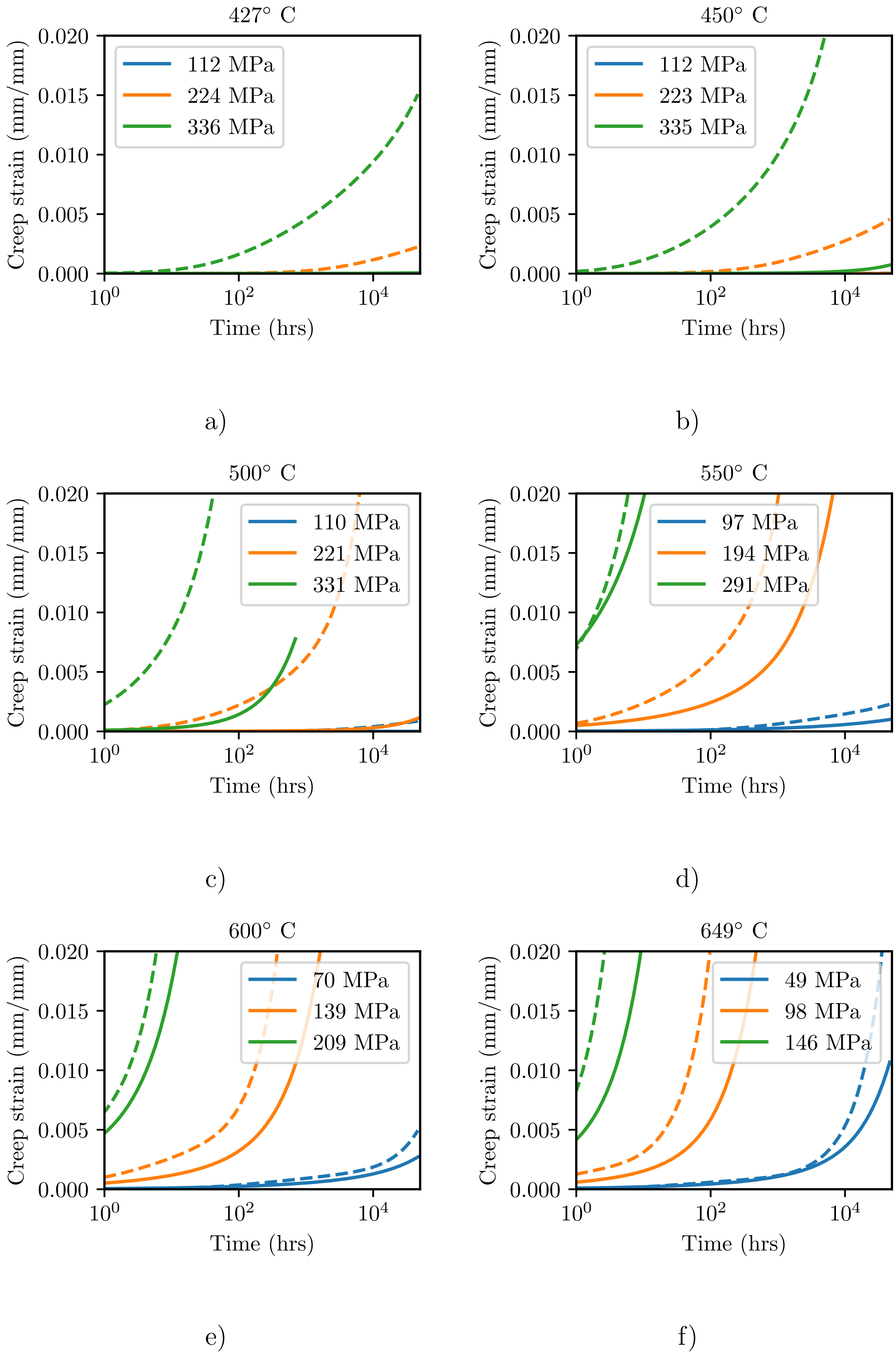

Figure 3.3: Comparison between the simple deformation model underlying the ASME isochronous curves and the full viscoplastic model on simulated creep tests. Curves not visible fall off the plot exhibit negligible creep.

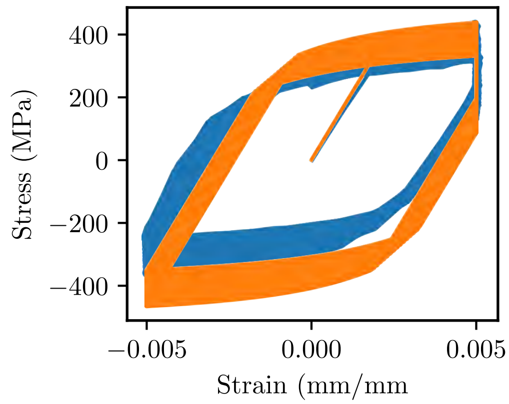

Figure 3.4: Example validation cyclic test. Test conditions are a 1% strain range, fully-reversed loading, $T = 550°$ C, and a 100 minute hold on the tensile end of the cycle. Experimental data is plotted in blue, model is plotted in orange.

3.3 Verifying implementations of the model

The formal definition of the model will be the Code definition of the mathematical model and parameters. Designers will then need to implement these mathematical relations in their finite element software by choosing an appropriate time integration scheme and programming the model. Such implementations will need to be verified. One option is to use the implementation provided by the report authors (`https://github.com/Argonne-National-Laboratory/neml`) to verify their implementation. Another option is to provide tabulated results that can be used to quickly check a particular implementation.

The model spans from room temperature to 650° C and is notionally valid for a wide range of strain rates. A relatively simple uniaxial cyclic simulation can be devised to test all the model features simultaneously. Figure 3.5 plots a particular set of strain-controlled cyclic input to the constitutive model by showing a single cycle of strain versus time and temperature versus time data. These plots are not to scale and that all transients are linear in time. Figure 3.6 then shows the resulting stress/strain and stress/time hysteresis data generated by running the model on this input. Table 3.2 tabulates the maximum and minimum stress in each cycle out to 5 repetitions and Figure 3.7 plots the same data. This table can be used to quickly verify a model implementation by running the load history defined in Figure 3.5 and comparing the results to the tabulated data. The simulation must include the appropriate thermal strains using the coefficient of thermal expansion in the model definition. In other words, the strain history defined in Figure 3.5 is showing total strain, not mechanical strain.

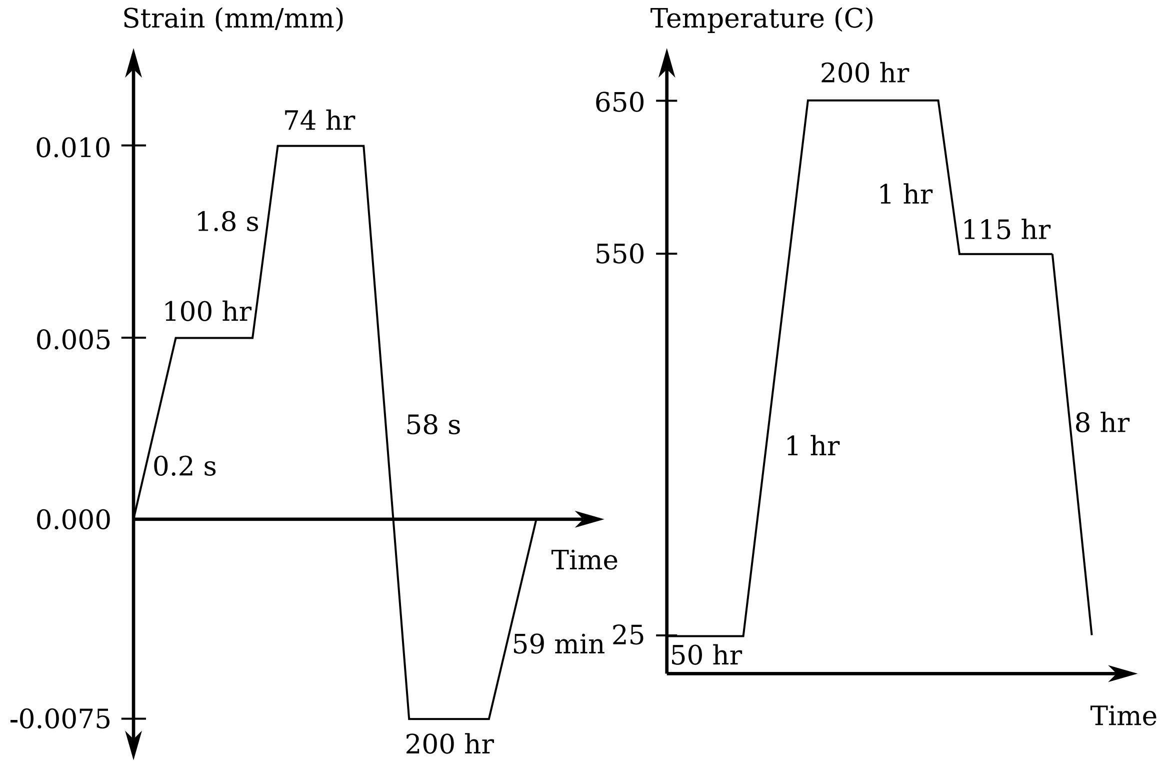

Figure 3.5: Verification cycle definition. Note the plots are not to scale and the total cycle period totals 375 hours. The validation test requires repeating this cycle 500 times.

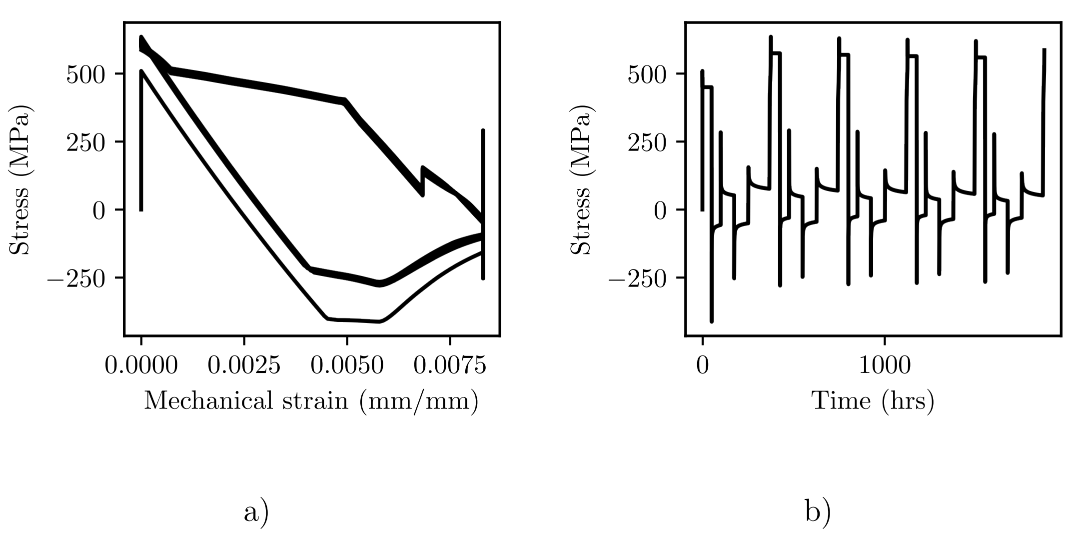

Figure 3.6: Results from the verification cycle plotted as a stress/mechanical strain hysteresis (a) and a stress/time history (b).

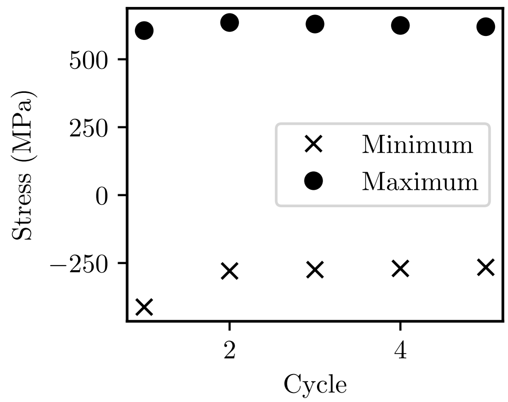

Figure 3.7: Maximum and minimum stress as a function of cycle count.

Cycle	Minimum stress (MPa)	Maximum stress (MPa)
1	-412.1	605.6
2	-279.2	635.3
3	-274.4	629.7
4	-269.9	624.6
5	-265.7	619.8

Table 3.2: Verification data: maximum and minimum stress over each cycle for the first five repetitions of the load.

4 Conclusions

This report describes a complete design by inelastic analysis method for Grade 91 steel proposed for incorporation into the ASME Boiler and Pressure Vessel Code Section III, Division 5, Subsection HB, Subpart B. The methodology contains guidance on inelastic modeling and a particular constitutive model for Grade 91 to be used with the existing ASME rules for design by inelastic analysis. Appendix A reproduces the draft Code language in full and this report will serve as the basis for a background document provided to ASME for balloting.

As discussed above, the proposed appendix will be balloted on the completion of both this work, represented by this report, and the development of a proposed inelastic model for 316H stainless steel under a separate work package. Completion of that model is expected by August 2019. Subsequently, the Code proposal will be balloted to the relevant Section III Codes and Standards committees starting with the Working Group on Inelastic Analysis Methods. This ballot will serve as a prototype for developing a process for balloting models for the remaining Class A materials.

A Current draft Appendix HBB-Z

The following text is the current draft Appendix HBB-Z on inelastic modeling produced for discussion in the ASME Boiler and Pressure Vessel Code committees.

NONMANDATORY APPENDIX HBB-Z
GUIDANCE ON MATERIAL MODELS FOR DESIGN BY INELASTIC ANALYSIS

HBB-Z-1100 INTRODUCTION

HBB-Z-1110 OBJECTIVE

The purpose of this nonmandatory appendix is to provide guidance on material models used for the design by inelastic analysis provisions of Section III, Division 5, Subsection HB, Subpart B (HBB), Nonmandatory Appendix HBB-T. Such models are used to analyze the component when designing against ratcheting strain accumulation (HBB-T-1200) and creep-fatigue damage (HBB-T-1400). This appendix provides general guidance on developing a material model adequate for use with Appendix HBB-T (HBB-Z-1200) along with a specific set of material models for the Class A materials deemed suitable for use with the Appendix HBB-T methods (HBB-Z-1300).

HBB-Z-1120 HBB REQUIREMENTS

HBB-3214.2 provides the requirements for inelastic analysis. This appendix expands upon the basic requirements described in HBB-3214.2.

HBB-Z-1130 SPECIFICATION OF AN INELASTIC MODEL

The guidance given in HBB-Z-1200 provides a general set of criteria to consider when developing or specifying an inelastic material model. The models described in HBB-Z-1300 are considered acceptable for use with the Appendix HBB-T design by inelastic analysis methods. Alternate inelastic material models can be used with justification in the Design Report per HBB-3214.2.

HBB-Z-1200 GUIDANCE ON CONSTRUCTING A MATERIAL MODEL

HBB-Z-1210 GENERAL GUIDANCE

As described in HBB-3214.2, an inelastic material model must represent the response of the material to time dependent, thermomechanical load. The material model must represent cyclic hardening or softening, as appropriate, primary creep, the effect of prior creep on subsequent plastic deformation, and the effect of prior plasticity on subsequent creep. The guidance in HBB-1210 applies to all materials, HBB-Z-1220 provides guidance for specific materials.

HBB-Z-1211 VALIDITY OF THE MATERIAL MODEL

The material model should be validated across the entire temperature range experienced by the component for all Level A, Level B, and Level C Service Loadings. If the material model is to be used to evaluate service conditions after Level D loading it must also be validated to cover these conditions. The material model must be validated for the strain rates experienced by the component in service. These strain rates cannot be determined prior to the development of the inelastic material model and so the designer should check the design analysis for all Service Loadings to verify the material model remains within the validated range of strain rates. A typical component will experience very slow strain rates during hold conditions, typically less than $10^{-6}\mathrm{s}^{-1}$ and will experience much faster strain rates during loading.

The Service Loadings may cause portions of the component to descend below the T_{max} temperatures given by Table HAA-1130-1. If so, the material model must accurately represent plastic deformation down to the lowest temperature experienced by the component in service. Below T_{max} the component will not accumulate creep deformation or damage but may accumulate plastic strain causing additional fatigue damage and ratcheting strain accumulation. Furthermore, the inelastic material model must ensure continuity into the low temperature regime in order to execute the required design analysis.

HBB-Z-1212 KEY FEATURES TO REPRESENT

HBB-Z-1212.1 CYCLIC RESPONSE

The design by inelastic analysis methods require an accurate representation of the material's response under cyclic thermomechanical loading. Considering a single material point, the model must accurately represent the cyclic temperature-stress-strain-time hysteresis undergone by a point under the loading provided by the surrounding material. In general, this loading is neither fully strain nor stress controlled but rather some mix of both, exemplified by a load including elastic follow up. A suitable inelastic constitutive model should therefore capture this hysteretic response for general thermomechanical cyclic load with the most important features being the strain range, the stress relaxation profile during any holds in the loading, and the accumulated ratcheting strain.

HBB-Z-1212.2 MATERIAL BATCH VARIATION

By default, a material model should capture the average response of all available heats of the particular Class A material. Even for material falling within the material specifications and the additional restrictions given in HBB-2000 there can be a wide range of material properties causing cyclic responses that differ greatly across multiple heats of material. The procedure used to develop the material model must account for this variability and represent the average behavior of all acceptable material.

HBB-Z-1212.3 ACCUMULATED DAMAGE

The material model must account for the interaction of creep and plasticity, including cyclic softening effects. However, some softening effects are caused by the development of damage in the material affecting its elastic properties. For example, during prolonged creep cavitation processes will open voids in the material leading to a loss of material strength and eventual failure.

For evaluating creep-fatigue damage (HBB-T-1400) the inelastic material model need not account for softening caused by development of damage in a material. It must however account for all other material softening mechanisms, for example those caused by rearrangements of the material's dislocation or grain structure. The rational is the Appendix HBB-T procedure for creep-fatigue design by inelastic analysis explicitly and separately accounts for the accumulation of damage in the material using the Code rules. As such, representing damage development in the inelastic material model double counts damage and may lead to an over conservative design. This means, for example, that an inelastic material model used to assess creep-fatigue damage using the Appendix HBB-T provisions need not represent tertiary creep.

However, an inelastic material model used for checking for strain accumulating using HBB-T-1200 must represent the development of damage in the material. Damage will tend to accelerate cyclic strain accumulation and so a model must represent the development of damage in order to accurately simulate ratcheting strain accumulation.

This means that two inelastic material models for a given material could be specified or developed: one for creep-fatigue damage and one for ratcheting strain accumulation. It is however conservative to use a single material model, representing damage accumulation, for both the ratcheting strain and creep-fatigue design checks. The models provided in HBB-Z-1300 are of this type.

HBB-Z-1212.4 COUPLED CREEP-PLASTICITY

A material model must account for the interaction of creep and plasticity in the material. At the very least, this implies a non-unified material model representing inelastic deformation through a combination of a rate independent plastic strain added to a rate dependent creep strain and must include a mechanism for the creep model to affect the plasticity model and vice-versa.

At sufficiently high temperatures creep and plasticity are inherently coupled and cannot be distinguished. Table HBB-Z-1212.4-1 provides guidance on this temperature threshold for the Class A materials. Above this threshold temperature non-unified, additive material models are unsuitable and the material model should have a unified viscoplastic formulation.

Table HBB-Z-1212.4-1

Material	Threshold temperature, °F (°C)
304 SS	1160 (625)
316 SS	1180 (640)
Ni-Fe-Cr (Alloy 800H)	1300 (710)
2¼Cr-1Mo	1020 (550)
9Cr-1Mo-V	840 (450)

HBB-Z-1212.5 STRESS MULTIAXIALITY

Actual components experience multiaxial stress states, while material model forms are commonly developed from uniaxial test data. However, an adequate material model must accurately represent the effect of multiaxial stress states on material deformation and documentation.

HBB-Z-1213 MODEL VALIDATION

HBB-Z-1212 enumerates the key features a model must capture. Generally, a model can be validated by comparison to cyclic tests that include hold periods and creep test data. Cyclic tests best represent the actual loading a component will see in service. However, actual material points in components experience neither pure load nor pure displacement controlled loading. Therefore, a model should be validated against both stress- and strain-controlled cyclic test data which are bounding cases for the loading actually experience in service. Generally, long-term holds out to realistic component hold times are infeasible in cyclic tests. Therefore, a model should also be validated against long-term creep test data in order to validate its response for realistic creep and stress-relaxation periods. These validation tests should span the required temperature and strain rate range, as described in HBB-Z-1211.

A model should also be validated against a test with a non-uniaxial stress state in order to assess the assumptions used to generalize 1D uniaxial test data into a 3D material model.

Actual components experience non-isothermal loading. Materials may have a temperature rate effect that cannot be sampled using a series of isothermal tests at different temperatures. As such, a material model should be validated against non-isothermal experiments, for example standard thermomechanical tests, spanning the temperature range outlined in HBB-Z-1211.

HBB-Z-1220 SPECIFIC MATERIAL ISSUES

HBB-Z-1221 304 SS

There are no specific material issues to be aware of for 304 SS.

HBB-Z-1222 316 SS

There are no specific material issues to be aware of for 316 SS.

HBB-Z-1223 Ni-Fe-Cr (Alloy 800H)

There are no specific material issues to be aware of for Ni-Fe-Cr (Alloy 800H).

HBB-Z-1224 2¼Cr-1Mo

2¼Cr-1Mo exhibits cyclic softening within the temperature range covered by Subpart HBB. A suitable inelastic model must account for this softening.

HBB-Z-1225 9Cr-1Mo-V

As described in HBB-3214.2, 9Cr-1Mo-V undergoes cyclic softening at relatively low temperatures and a non-unified material model of creep-plasticity does not adequately capture the material's inelastic response. Table HBB-Z-1212.4-1 reflects coupling between creep and plasticity with a very low value of the threshold temperature.

HBB-Z-1300 ACCEPTABLE MATERIAL MODELS

HBB-Z-1310 INTRODUCTION

Designers may use the reference material models described in HBB-Z-1300 for the inelastic stress analysis required for fulfilling the Section III, Division 5, Subsection HB, Subpart B, Nonmandatory Appendix HBB-T design by inelastic analysis provisions.

HBB-Z-1320 MATERIAL MODELS

HBB-Z-1321 304 SS

The material model for 304 SS is under preparation.

HBB-Z-1322 316 SS

The material model for 316 SS is under preparation.

HBB-Z-1323 Ni-Fe-Cr (Alloy 800H)

The material model for Ni-Fe-Cr (Alloy 800H) is under preparation.

HBB-Z-1324 2¼Cr-1Mo

The material model for 2¼Cr-1Mo is under preparation.

HBB-Z-1325 9Cr-1Mo-V

The reference model for 9Cr-1Mo-V is defined by the following equations in rate form. The implementation of the model will require selecting an appropriate numerical time integration scheme. Table HBB-Z-1325-1 lists the notation used in HBB-Z-1325. Variables indicated by "rate" in the "Type" column comprise the rate-form definition of the model. Variables indicated by "history" in the "Type" column are internal history variables maintained by the model. Variables indicated by "parameter" in the "Type" column are model parameters, defined in Tables HBB-Z-1325-2 and HBB-Z-1325-3. Variables indicated by "descriptive" in the "Type" column are neither model parameters nor history variables and are used in the model exposition.

Table HBB-Z-1325-1

Variable	Units	Description	Type
$\boldsymbol{\sigma}$	MPa	Stress	descriptive
$\dot{\boldsymbol{\sigma}}$	MPa/s	Stress rate	rate
$\dot{\boldsymbol{\varepsilon}}$	1/s	Strain rate	descriptive
T	°C	Temperature	descriptive
$\dot{T}$	°C/s	Temperature rate	descriptive
E	MPa	Young's modulus	parameter
ν	-	Poisson's ratio	parameter
$\boldsymbol{C}$	MPa	Elasticity tensor	descriptive
α	1/°C	Coefficient of thermal expansion	parameter
$\boldsymbol{I}$	-	Identity tensor	descriptive
$\dot{\boldsymbol{\varepsilon}}_{inelastic}$	1/s	Inelastic strain rate	descriptive
k	mJ/°C	Boltzmann constant	parameter
μ	MPa	Shear modulus	descriptive
$\dot{\varepsilon}_0$	1/s	Reference strain rate	parameter
$\dot{\varepsilon}_{eff}$	1/s	Effective strain rate	descriptive
b	mm	Burgers vector	parameter
g	-	Kocks-Mecking parameter	descriptive
g_0	-	Rate dependent/independent transition	parameter
A	-	Rate sensitivity slope	parameter
B	-	Rate sensitivity intercept	parameter
C	-	Rate independent flow strength	parameter
$\dot{\boldsymbol{\varepsilon}}_{ri}$	1/s	Rate independent inelastic rate	descriptive
$\dot{\boldsymbol{\varepsilon}}_{rd}$	1/s	Rate dependent inelastic rate	descriptive
f	MPa	Yield and flow function	descriptive
$\dot{\gamma}$	1/s	Plastic multiplier	descriptive
n	-	Rate sensitivity exponent	descriptive
η	$MPa/s^{1/n}$	Viscoplastic fluidity	descriptive
σ_0	MPa	Threshold stress	descriptive
σ_1	MPa	Isotropic hardening	descriptive
$\boldsymbol{x}$	MPa	Kinematic hardening	descriptive
$\dot{\sigma}_1$	MPa/s	Isotropic hardening rate	rate
δ	-	Voce parameter	parameter
Q	MPa	Voce saturation stress	parameter
$\boldsymbol{x}_1$	MPa	First backstress	descriptive
$\dot{\boldsymbol{x}}_1$	MPa/s	First backstress rate	rate
$\boldsymbol{x}_2$	MPa	Second backstress	descriptive
$\dot{\boldsymbol{x}}_2$	MPa	Second backstress rate	rate
C_1	MPa	First Chaboche hardening parameter	parameter
γ_1	-	First Chaboche dynamic recovery parameter	parameter
S_1	MPa^{1-s_1}	First Chaboche static recovery prefractor	parameter
s_1	-	First Chaboche static recovery prefractor	parameter
C_1	MPa	Second Chaboche hardening parameter	parameter
γ_1	-	Second Chaboche dynamic recovery parameter	parameter
S_2	MPa^{1-s_2}	Second Chaboche static recovery prefractor	parameter
s_2	-	Second Chaboche static recovery prefractor	parameter
p	MPa	Pressure	descriptive
$\boldsymbol{s}$	MPa	Stress deviator	descriptive
h	-	Stress measure parameter	parameter
l	-	Stress measure parameter	parameter

This material model uses metric units with temperature in Celsius and the definition here assumes a small strain constitutive response. The following definition of the stress rate and the rate of each history variable fully defines the material model. An implementation then numerically integrates the rate form definition. The material model as presented here is given in strain-space, where the input is the strain, strain rate, temperature, and temperature rate and the output is the stress rate. All logarithms in the subsequent definitions are natural logarithms.

The material model is defined by the rate equations:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{C}:\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_{inelastic} - \alpha\dot{T}\boldsymbol{I}\right)$$

$$\dot{\sigma}_1 = \delta(\sigma_1 - Q)\dot{\gamma}$$

$$\dot{\boldsymbol{x}}_1 = \left(\frac{2}{3}C_1\frac{\partial f}{\partial\boldsymbol{\sigma}} - \sqrt{\frac{2}{3}}\gamma_1\boldsymbol{x}_1\right)\dot{\gamma} - S_1\sqrt{\frac{3}{2}}\|\boldsymbol{x}_1\|^{s_1-1}\boldsymbol{x}_1$$

$$\dot{\boldsymbol{x}}_2 = \left(\frac{2}{3}C_2\frac{\partial f}{\partial \boldsymbol{\sigma}} - \sqrt{\frac{2}{3}}\gamma_2 \boldsymbol{x}_2\right)\dot{\gamma}$$

where : indicates double contraction of the elasticity tensor on the strain rate. The elasticity tensor is isotropic, defined by the temperature-dependent values of E and ν given in Section II, Part D (Metric) Tables TM-1 and PRD. Section II, Part D (Metric) Table TE-1 defines the values of the temperature dependent instantaneous coefficient of thermal expansion.

The flow surface is

$$f = \sqrt{\frac{3}{2}(\boldsymbol{s}-\boldsymbol{x}):(\boldsymbol{s}-\boldsymbol{x})} - h\,\mathrm{sign}(p)p^l - \sqrt{\frac{2}{3}}(\sigma_0+\sigma_1)$$

where $\boldsymbol{s}$ is the stress deviator

$$\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}\,\boldsymbol{\sigma}\,\boldsymbol{I}$$

with tr indicating the trace of a tensor, the double-brackets indicating $\|\boldsymbol{Y}\| = \sqrt{\boldsymbol{Y}:\boldsymbol{Y}}$ with : indicating double contraction on the tensor, and

$$\boldsymbol{x} = \boldsymbol{x}_1 + \boldsymbol{x}_2$$

The inelastic strain rate, plastic multiplier, and definition of the threshold stress depend on the applied strain rate and temperature:

$$\dot{\boldsymbol{\varepsilon}}_{inelastic} = \begin{cases}\dot{\boldsymbol{\varepsilon}}_{ri} & g \le g_0\\ \dot{\boldsymbol{\varepsilon}}_{rd} & g > g_0\end{cases}$$

with

$$g = \frac{k(T+273.15)}{\mu b^3}\log\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_{eff}}\right)$$

with

$$\dot{\varepsilon}_{eff} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}:\dot{\boldsymbol{\varepsilon}}}$$

with : again indicating double contraction and the shear modulus defined in terms of the Young's modulus and Poisson's ratio:

$$\mu = \frac{E}{2(1+\nu)}$$

The strain rate and plastic multiplier are given by a standard rate-independent update defined by the Kuhn-Tucker and consistency conditions for the rate independent case ($g \le g_0$):

$$\dot{\boldsymbol{\varepsilon}}_{ri} = \dot{\gamma}\frac{\partial f}{\partial \boldsymbol{\sigma}}$$
$$\dot{\gamma} > 0$$
$$f \le 0$$
$$\gamma f = 0$$
$$\gamma \dot{f} = 0$$

The strain rate and plastic multiplier are given by a Perzyna viscoplastic update for the rate dependent case ($g > g_0$)

$$\dot{\boldsymbol{\varepsilon}}_{rd} = \dot{\gamma}\frac{\partial f}{\partial \boldsymbol{\sigma}}$$
$$\dot{\gamma} = \sqrt{\frac{3}{2}}\langle\frac{f}{\sqrt{2/3}\eta}\rangle^n$$
$$n = -\frac{\mu b^3}{kTA}$$
$$\eta = e^B\mu\dot{\varepsilon}_0^{-1/n}$$

The threshold stress likewise has a different definition in the rate dependent and rate independent regimes

$$\sigma_0 = \begin{cases}\mu e^C & g \le g_0\\ 0 & g > g_0\end{cases}$$

These equations completely describe the model. Table HBB-Z-1325-2 gives the temperature independent model parameters. Table HBB-Z-1325-3 gives the temperature dependent parameters, which should be interpolated linearly between the provided temperature values, with the exception of parameters S_1 and S_2 which should be interpolated log-linearly.

Table HBB-Z-1325-2

Parameter	Value	Units
k	1.38068e-20	mJ/°C
$\dot{\varepsilon}_0$	1e10	1/s
b	2.48e-7	mm
g_0	0.3496	-
A	-9.698	-
B	-8.509	-
C	-5.119	-

Table HBB-Z-1325-3

Parameter	Units	25°C	400°C	500°C	550°C	600°C	650°C
h	-	2e-4	2e-4	2e-4	2e-4	2e-4	2e-4
l	-	1.91	1.91	1.71	1.69	1.61	1.51
Q	MPa	-96	-96	-150	-151	-151	-131
δ	-	2.00	1.71	1.71	1.51	1.51	1.00
C_1	MPa	14500	15000	19000	19200	19900	19000
γ_1	-	141	141	802	792	803	803
S_1	MPa^{1-s_1}	1e-15	1e-15	1e-15	1e-15	1e-15	1e-15
s_1	-	3.5	3.5	5.97	5.97	7.47	9.46
C_2	MPa	12500	12500	12500	12600	12400	12400
γ_2	-	60.6	60.4	200	200	202	020
S_2	MPa^{1-s_2}	1e-15	1e-15	1e-15	1e-15	1e-15	1e-15
s_2	-	3.5	3.5	5.96	5.96	7.51	9.53

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Bibliography

[1] M. C. Messner, V.-T. Phan, and T.-L. Sham, “FY17 Status Report on the Initial Development of a Constitutive Model for Grade 91 Steel,” tech. rep., Argonne National Laboratory ANL-ART-93 OSTI 136734, 2017.

[2] M. C. Messner, V.-T. Phan, and T.-L. Sham, “Development of Grade 91 inelastic model for incorporation in ASME Division 5,” tech. rep., Argonne National Laboratory ANL-ART-137 OSTI 146678, 2018.

[3] M. C. Messner, V.-T. Phan, and T.-L. Sham, “A Unified Inelastic Constitutive Model for the Average Engineering Response of Grade 91 Steel,” in *Proceedings of the 2018 ASME Pressure Vessels and Piping Conference*, PVP2018-84104, 2018.

[4] A. K. Dhalla, “Recommended practices in elevated temperature design: a compendium of breeder reactor experiences (1970-1987): Volume III - Inelastic analysis,” Tech. Rep. 366, 1991.

[5] J. M. Corum, “Evaluation of Inelastic Analysis Methods,” in *Transactions of the 4th International Conference on Structural Mechanics in Reactor Technology*, 1977.

[6] K. C. Liu, “Room-Temperature Elastic-Plastic Response of Thin-Walled Tubes Subjected to Nonradial Combinations of Axial and Torsional Loadings,” in *Pressure Vessel and Piping: Verification and Qualification of Inelastic Analysis Computer Programs* (J. M. Corum and W. B. Wright, eds.), New York: ASME Press, 1975.

[7] C. E. Pugh, “Constitutive Equations for Creep Analysis of LMFBR Components,” in *Advances in Design for Elevated Temperature Environment* (S. Y. Zamrik and R. I. Jetter, eds.), New York: ASME Press, 1975.

[8] C. E. Pugh and D. N. Robinson, “Some Trends in Constitutive Equation Model Development for High-Temperature Behavior of Fast-Reactor Structural Alloy,” *Nuclear Engineering and Design*, vol. 48, pp. 269–276, 1978.

[9] C. E. Pugh and D. N. Robinson, “Constitutive Equations for Meeting Elevated Temperature Design Needs,” in *Pressure Vessel and Piping Design Technology, A Decade of Progress*, pp. 7–10, 1981.

[10] DS Simula, “4.3.8 ORNL constitutive theory,” in *Abaqus Theory Manual*, 2016.

[11] J. L. Chaboche, “A review of some plasticity and viscoplasticity constitutive theories,” *International Journal of Plasticity*, vol. 24, pp. 1642–1693, oct 2008.

[12] D. N. Robinson and R. W. Swindeman, “Unified Creep-Plasticity Constitutive Equations for 2-1/4Cr-1Mo Steel at Elevated Temperature,” tech. rep., Oak Ridge National Laboratory, ORNL/TM-8444, 1982.

[13] S. Kim and J. R. Weertman, “Investigation of Microstructural Changes in a Ferritic Steel Caused by High Temperature Fatigue,” *Metallurgical Transactions*, vol. 19A, no. April, pp. 999–1007, 1988.

[14] V.-T. Phan, M. C. Messner, and T.-L. Sham, "A Unified Engineering Inelastic Model for 316H Stainless Steel," in *Proceedings of the 2019 ASME Pressure Vessels and Piping Conference*, 2019.

[15] J. R. Stefano, V. K. Sikka, J. J. Blass, C. R. Brinkman, J. M. Corum, J. A. Horsk, R. L. Huddlestor, J. F. King, R. W. McClung, and W. K. Sartory, "Summary of Modified 9Cr-1Mo Development Program: 1975-1985," tech. rep., Oak Ridge National Laboratory, 1986.

[16] M. C. Messner, R. I. Jetter, and T.-L. Sham, "Establishing Temperature Upper Limits for the ASME Section III, Division 5 Design by Elastic Analysis Methods," in *ASME 2018 Pressure Vessels and Piping Conference*, pp. V01BT01A016–V01BT01A016, 2018.

[17] M. C. Messner and T.-L. Sham, "Development of ASME Division 5 Code proposal on temperature limits for simplified design methods," tech. rep., Argonne National Laboratory, ANL-ART-132, OSTI 146672, 2018.

[18] J. L. Chaboche and G. Cailletaud, "On the calculation of structures in cyclic plasticity or viscoplasticity," *Computers and Structures*, vol. 23, no. 1, pp. 23–31, 1986.

[19] J. L. Chaboche, "Time-independent constitutive theories for cyclic plasticity," *International Journal of Plasticity*, vol. 2, no. 2, pp. 149–188, 1986.

[20] J. Chaboche, "Constitutive equations for cyclic plasticty and cyclic viscoplasticity," *International Journal of Plasticity*, vol. 5, pp. 247–302, 1989.

[21] J. L. Chaboche, "On some modifications of kinematic hardening to improve the description of ratchetting effects," *International Journal of Plasticity*, vol. 7, no. 7, pp. 661–678, 1991.

[22] J. L. Chaboche and G. Cailletaudb, "Integration methods for complex plastic constitutive equations," *Computational Methods in Applied Mechanics and Engineering*, vol. 133, no. 95, pp. 125–155, 1996.

[23] J.-L. Chaboche, "Thermodynamic formulation of constitutive equations and application to the viscoplasticity and viscoelasticity of metals and polymers," *International Journal of Solids and Structures*, vol. 34, no. 18, pp. 2239–2254, 1997.

[24] J. L. Chaboche and O. Jung, "Application of a kinematic hardening viscoplasticity model with thresholds to the residual stress relaxation," *International Journal of Plasticity*, vol. 13, no. 10, pp. 785–807, 1998.

[25] M. Yaguchi and Y. Takahashi, "Ratchetting of viscoplastic material with cyclic softening, part 1: Experiments on modified 9Cr-1Mo steel," *International Journal of Plasticity*, vol. 21, no. 1, pp. 43–65, 2005.

[26] M. Yaguchi and Y. Takahashi, "Ratchetting of viscoplastic material with cyclic softening, part 2: application of constitutive models," *International Journal of Plasticity*, vol. 21, pp. 835–860, 2005.

[27] H. Mecking and U. F. Kocks, "Kinetics of flow and strain-hardening," *Acta Metallurgica*, vol. 29, pp. 1865–1875, 1981.

[28] H. Mecking, B. Nicklas, N. Zarubova, and U. F. Kocks, "A "universal" temperature scale for plastic flow," *Acta Metallurgica*, vol. 34, no. 3, pp. 527–535, 1986.

[29] T. Asayama and Y. Tachibana, *Existing Evaluation Procedures for Grade 91 and Hastelloy XR.* 2009.

[30] B. K. Choudhary and E. Isaac Samuel, "Creep behaviour of modified 9Cr-1Mo ferritic steel," *Journal of Nuclear Materials*, vol. 412, no. 1, pp. 82–89, 2011.

[31] K. Kimura, H. Kushima, and K. Sawada, "Long-term creep deformation property of modified 9Cr-1Mo steel," *Materials Science and Engineering A*, vol. 510-511, no. C, pp. 58–63, 2009.

[32] G. H. Koo and J. H. Lee, "Investigation of ratcheting characteristics of modified 9Cr-1Mo steel by using the Chaboche constitutive model," *International Journal of Pressure Vessels and Piping*, vol. 84, no. 5, pp. 284–292, 2007.

[33] G. H. Koo and J. H. Kwon, "Identification of inelastic material parameters for modified 9Cr-1Mo steel applicable to the plastic and viscoplastic constitutive equations," *International Journal of Pressure Vessels and Piping*, vol. 88, no. 1, pp. 26–33, 2011.

[34] S. Latha, M. Nandagopal, S. P. Selvi, K. Laha, and M. D. Mathew, "Tensile and creep behaviour of modified 9Cr-1Mo steel cladding tube for fast reactor using metallic fuel," *Procedia Engineering*, vol. 86, pp. 71–79, 2014.

[35] K. Maruyama, J. Nakamura, K. Yoshimi, and Y. Nagae, "Evaluation of long-term creep rupture life of Gr.91 steel by analysis of on-going creep curves," in *Proceedings from the Eighth International Conference on Advances in Materials Technoglogy for Fossil Power Plants*, pp. 467–478, 2016.

[36] R. W. Swindeman, "Cyclic Stress-Strain-Time Response of a 9CMMo-V-Nb Pressure Vessel Steel at High Temperature," in *Low Cycle Fatigue, ASTM STP 942* (H. D. Solomon, G. R. Halford, L. R. Kaisand, and B. N. Leis, eds.), pp. 107–122, Philadelphia: American Society for Testing and Materials, 1988.

[37] M. Yaguchi and Y. Takahashi, "Unified Inelastic Constitutive Model for Modified 9Cr-1Mo Steel Incorporating Dynamic Strain Aging Effect," *JSME International Journal, Series A*, vol. 42, no. 1, pp. 1–10, 1999.

[38] K. Zhang and J. Aktaa, "Characterization and modeling of the ratcheting behavior of the ferritic-martensitic steel P91," *Journal of Nuclear Materials*, vol. 472, pp. 227–239, 2016.

[39] R. W. Swindeman, "Construction of isochronous stress-strain curves for 9Cr-1Mo-V Steel," *Advances in Life Prediction Methodology*, vol. 391, pp. 95–100, 1999.

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